Exam 2 will be returned tomorrow

Closing *Tues*: 4.3

Closing *Thurs*: 4.4

Closing next *Tues*: 4.4-5

Closing next *Thurs*: 4.7 (last assignment)

I strongly suggest you finish 4.5 by the end of this week and so you can devote the last week to 4.7 and final studying.

4.3 Local Max/Min and 1st and 2nd derivative tests *(continued)*

Entry Task:

Find and classify the critical points for

$$y = 2 + 2x^2 - x^4$$

(*use the 1st deriv. test*)

The 2nd Derivative

$$y'' = f''(x) = \frac{d}{dx}(f'(x))$$

= "rate of change of 1st deriv."

Terminology

If **f''(x) is positive**,
then the **slope of f(x) is *increasing***
and we say f(x) is **concave up**.

If **f''(x) is negative**,
then the **slope of f(x) is *decreasing***
and we say f(x) is **concave down**.

A point in the domain of the function at which the concavity changes is called an **inflection point**.

Summary:

$\boldsymbol{y = f(x)}$	$\boldsymbol{y'' = f''(x)}$
possible inflection	zero
concave up	positive
concave down	negative
possible inflection	does not exist

Example: Find all inflection points and indicate where the function is concave up and concave down for

$$y = x^4 - 2x^3$$

Clever Observation

(*Second Derivative Test*)

If $x = a$ is a critical number for $f(x)$

AND

1. if $f''(a)$ is positive (CCU),
   then a local min occurs at $x = a$.

2. if $f''(a)$ is negative (CCD),
   then a local max occurs at $x = a$.

3. if $f''(a) = 0$,
   then we say the 2nd deriv. test is
   inconclusive (need other method)

Example: Find and classify the critical numbers for

$$y = 2 + 2x^2 - x^4$$

(use the 2^{nd} deriv. test)

4.4 L'Hopital's Rule

First, recall as we discussed many, many, many times at the beginning of the term:

(Assuming f and g are cont. at *x=a*)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = ??$$

- If $g(a) \neq 0$, then done!

  Ans = $\frac{f(a)}{g(a)}$.

- If $g(a) = 0$ and $f(a) \neq 0$, then examine each side of *x = a* (look at the signs)

  Ans = $\infty, -\infty$, or DNE.

- If $g(a) = 0$ and $f(a) = 0$, then use algebra to rewrite and 'cancel' the denominator.

L'Hopital's Rule (0/0 case)

Suppose $g(a) = 0$ and $f(a) = 0$ and *f* and *g* are differentiable at *x* = *a*, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Examples:

1. $\lim_{x \to 4} \dfrac{16 - x^2}{4 - x}$

2. $\lim_{x \to 0} \dfrac{\sin(x)}{x}$

3. $\lim_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{x}$

Aside: Sketch of derivation

Assume $g(a) = 0$ and $f(a) = 0$

(These explanations are for the case when g'(a) is not zero).

Explanation 1 (def'n of derivative)

$$\frac{f'(a)}{g'(a)} = \frac{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}}{\lim_{x \to a} \frac{g(x) - g(a)}{x - a}}$$

provided these limits exist we have:

$$\frac{f'(a)}{g'(a)} = \lim_{x \to a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x \to a} \frac{f(x)}{g(x)}$$

Explanation 2 (tangent line approx.):

The tangent lines for $f(x)$ and $g(x)$ at $x = a$ are

$$y = f'(a)(x - a) + 0$$

$$y = g'(a)(x - a) + 0$$

And we know these approximate the functions f(x) and g(x) better and better the closer x gets to a, so

Thus,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(a)(x - a)}{g'(a)(x - a)} = \frac{f'(a)}{g'(a)}$$